SIMPLIFIED ANALYSIS OF NON-EQUILIBRIUM SEMICONDUCTOR SPACE CHARGE REGIONS USING A ZERO-CURRENT APPROXIMATION

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ABSTRACT

An explicit zero-current approximation is introduced to derive simple one-dimensional expressions for the semiconductor potential, electric field and carrier concentrations in a non-equilibrium MIS structure which include the effect of the potential drop across the minority carrier inversion layer. The expressions are an alternative to more refined but complex models of semiconductor space-charge regions in which a current flows. The zero-current approximation is particularly useful for high capacitance thin insulator MIS structures.

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INTRODUCTION

The relationships among parameters describing the state of a semiconductor surface such as band-bending, semiconductor space charge, and carrier concentration profiles have been well established for equilibrium conditions [1-3]. Treatment for non-equilibrium conditions in a metal/ insulator/ semiconductor (MIS) device structures such as MIS tunnel diodes and charge-coupled devices has been covered in more recent works [4-10]. These latter contributions have either avoided the potential drop across the surface minority carrier inversion layer or have involved complex numerical computation.

In this brief, a zero-current approximation is introduced which simplifies the analytical complexity and yields expressions which are easily evaluated numerically. We believe that the approximation used is intuitively satisfying and yields insight into the important aspects of this otherwise complex physical system, and have found it to be useful in our modeling studies of the bistable metal/tunnel-oxide/semiconductor junction [11,12].

ANALYSIS

In this analysis, an MIS structure on an n-type substrate (uniform doping) is assumed, though the results are valid for a p-type substrate with the appropriate sign reversals. The current density equation for the minority carriers (holes) is:

$$J_p = q\mu_p pE - qD_p(dp/dx) \tag{1}$$

where E is the electric field, and p is the hole concentration. In the case where the net current is much smaller than either component of Eq. 1, then it is approximately correct to write

$$d(\ln p)/dx = -(q/kT)\cdot(dU/dx) \tag{2}$$

such that U is the semiconductor potential (negative for a depleted n-type

substrate) and where the Einstein relationship between mobility (μ_p) and diffusion coefficient (D_p) has been employed. This is not to say that the net current need be miniscule, rather, it is a statement that a sizeable net current can be accounted for by a slight adjustment of carrier concentration profiles. Integrating this equation from the surface where $U = U_s$ to a distance x from the oxide/semiconductor interface yields

$$p(x) = p_s \exp[\beta(U_s - U(x))] \tag{3}$$

where p_s is the surface minority carrier concentration, and β is defined as q/kT. A similar analysis for electrons yields

$$n(x) = n_{no} \exp[\beta U(x)] \tag{4}$$

where n_{no} is the bulk majority carrier concentration for $U = 0$. Writing these carrier concentrations in this manner is the essence of the zero-current approximation, and is equivalent to an a priori assumption of flat quasi-Fermi levels.

The semiconductor space charge density $\rho(x)$ is given by

$$\rho(x) = q[p(x) - n(x) - p_{no} + n_{no}] \tag{5}$$

where p_{no} is the bulk minority carrier concentration, and p(x) and n(x) are given by Eqs. 3 and 4 above. This expression for the space charge density may be substituted into Poisson's equation:

$$d^2U/dx^2 = -\rho/\varepsilon_s\varepsilon_o \tag{6}$$

where ε_o is the permittivity of free space and ε_s is the relative dielectric constant of the semiconductor, and integrated following the method of Kingston and Neustadter [1] to yield:

$$E(x) = -\left[(2q/\beta\varepsilon_s\varepsilon_o) \cdot [n_{no}(e^{\beta U(x)} - \beta U(x) - 1) + p_s(e^{\beta U_s} - \beta U(x) + \frac{p_{no}}{p_s}\beta U(x) - e^{\beta U_s})] \right]^{1/2} \tag{7}$$

In particular, the electric field at the surface is given by:

$$E_s = -\left[(2q/\beta\varepsilon_s\varepsilon_o)\cdot[n_{no}(e^{\beta U_s} - \beta U_s - 1) + p_s(1 + \frac{p_{no}}{p_s}\beta U_s - e^{\beta U_s})]\right]^{1/2} \quad (8)$$

For an MIS structure without voltage dependent interface trap charge, the gate voltage V_g is related to the surface potential U_s and surface electric field according to

$$V_g - V_{fb} = U_s + (\varepsilon_s/\varepsilon_{in})E_s d_{in} \quad (9)$$

where V_{fb} is the flat band voltage, ε_{in} is the relative dielectric constant of the insulator, and d_{in} is the insulator layer thickness. Depending upon the application, Eqs. 8 and 9 may be evaluated in different ways. For example, if the gate voltage and insulator voltage drop are fixed, then Eq. 9 may be used to determine U_s, followed by the use of Eq. 8 to determine the surface minority carrier concentration p_s.

In the case of deep depletion (pn product smaller than the square of the intrinsic carrier concentration n_i) or equilibrium inversion ($pn = n_i^2$), $|\beta U_s|$ is generally much greater than unity as is $\exp[-\beta U_s]$, so that Eq. 8 is reduced to

$$E_s = -[(2q/\beta\varepsilon_s\varepsilon_o)\cdot(p_s - n_{no}\beta U_s)]^{1/2} \quad (10)$$

Solution of this equation for p_s yields

$$p_s = (\beta/2q\varepsilon_s\varepsilon_o)Q_s^2 + n_{no}\beta U_s \quad (11)$$

where the total semiconductor space charge Q_s is given by Gauss' Law as

$$Q_s = \varepsilon_s\varepsilon_o E_s \quad (12)$$

In equilibrium, p_s must also satisfy

$$p_s = p_{no}\exp[-\beta U_s] \quad (13)$$

Equation 11 agrees with Sze [10] only if the depletion approximation is used for U_s. For large values of inversion layer charge (as for a very thin insulator high capacitance MIS structure), the failure of the depletion

approximation to account for the voltage drop across the inversion layer becomes significant. The potential drop across the inversion layer δU can be estimated using Eq. 3 with $p(x) = n_{no}$ so that

$$\delta U = (kT/q)\cdot\ln(p_s/n_{no}) \tag{14}$$

and can exceed a hundred millivolts.

Once Eqs. 8 and 9 have been used to establish the surface values E_s and U_s, a simple algorithm can be used to determine $U(x)$ from the surface to the bulk. Using an incremental value of potential ΔU typically equal to $1/10\beta$, a corresponding distance Δx can be determined using

$$\Delta x = -\Delta U/E(x) \tag{15}$$

$E(x+\Delta x)$ may be determined from Eq. 7 using

$$U(x+\Delta x) = U(x) + \Delta U \tag{16}$$

In this manner, the potential profile may be determined by working backwards into the bulk from the surface. In addition, $p(x)$ and $n(x)$ may be determined using Eqs. 3 and 4. It should be noted that the zero-current approximation yields a good estimate of $U(x)$ through $\rho(x)$, but the nature of the approximation results in a poor estimate of the minority carrier concentration $p(x)$ near the depletion region edge and of the majority carrier concentration $n(x)$ near the surface.

CONCLUSIONS

Some care must be exercised when utilizing these expressions for large values of p_s since they have been derived for a charged gas in a charged lattice. The local density of states and energy distribution of the carriers has been ignored (except through the use of n_i) and may be important in some applications. Quantum well effects at the surface should also be considered. The latter results in an increased minority carrier charge moment and an

increased potential drop across the inversion layer. Nevertheless, we have used these equations, for example, to describe the relationship between inversion layer areal charge density and depletion layer depth, and the agreement between the model and experiment has been quite good.

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