

Application of Photon Statistics to the Quanta Image Sensor

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Abstract—The Poisson statistics of photon arrival rates are applied to the imaging performance of the Quanta Image Sensor (QIS) concept. Signal and noise as a function of exposure is analyzed and we find SNR only obeys the square-root law under sparse exposure. The D-log H characteristic of the QIS is quantified. Linearity and dynamic range are also investigated.

I. SIGNAL

Photons are emitted from light sources at some average rate on longer time scales, but randomly on shorter time scales and their emission from most light sources is well-described by the Poisson process. Through various attenuating and reflective processes, the photons arrive at a photodetector where they are absorbed and converted to photoelectrons and collected with some quantum efficiency (QE). Both the photon stream entering the photodetector and the photoelectrons collected by the photodetector are described by the Poisson process. This means that the average number of arrivals over some time interval τ depends only on the average arrival rate $\bar{\phi}$ (e.g. photoelectrons per second per photodetector) and the length of the interval τ , and in fact, just on the product $\bar{\phi}\tau$ that we will call the *quanta exposure* H . A quanta exposure of H means H photons or photoelectrons arrive at the photodetector on average over the interval τ .

In the Poisson process, the probability $P[k]$ of k arrivals in the interval τ with average arrival rate $\bar{\phi}$ is given by [1]:

$$P[k] = \frac{e^{-H} H^k}{k!} \quad (1)$$

Thus, the probability of no arrivals ($k=0$) is simply:

$$P[0] = e^{-H} \quad (2)$$

The probability of at least one arrival ($k>0$) is simply given by:

$$P[k > 0] = 1 - P[0] = 1 - e^{-H} \quad (3)$$

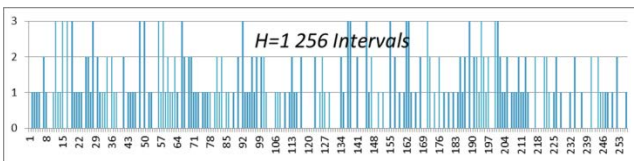


Fig. 1. A Monte-Carlo simulation of arrivals per interval is shown below for $H=1$ for 256 intervals. It is seen that many intervals have multiple arrivals and some have no arrivals.

The Quanta Image Sensor (QIS) [2] is organized as an array of photodetectors each sensitive to a single photoelectron. While in practice the photodetector may continue to be sensitive to multiple photoelectrons,

subsequent circuitry discriminates the output to two binary states: either a “0” meaning no photoelectron, or a “1” meaning at least one photoelectron. These highly specialized photodetectors are referred to as “jots.” [3] The QIS belongs to a class of sensors that could be called photon-counting or photoelectron-counting sensors and several papers have addressed photon statistics with regard to this class of sensor. [4-6]

Each jot has an integration period τ during which one or more photoelectrons might be collected. After the end of the integration period, the state of the jot is read out. It is then reset and the process starts again, typically with the same integration period τ . Note that the integration period τ could be less than the time between readouts since the jot could be reset at some time between readout cycles.

Let the jot have just two states at the end of the integration period, J_0 or J_1 , corresponding respectively to the absence or presence of at least one photoelectron. The probabilities of these states $P[J_0]$ and $P[J_1]$ are given by equations (2) and (3) respectively.

In an ensemble of M jots uniformly illuminated, let the expected number of jots in state J_0 be given by M_0 :

$$M_0 = M \cdot P[J_0] = M e^{-H} \quad (4)$$

and the expected number of jots in state J_1 be given by M_1 :

$$M_1 = M \cdot P[J_1] = M \cdot [1 - e^{-H}] \quad (5)$$

For sparse exposure ($H \lesssim 0.1$), the expected number of jots in the state J_1 is given by the linear relationship:

$$M_1 \cong MH \quad (6)$$

The J_1 density (M_1/M) at full exposure¹ ($H=1$) is just 63%. At 2x overexposure ($H=2$), it is 86%, and at 5x overexposure ($H=5$) the density reaches 99.3%. Density is shown in Fig. 2 as a function exposure.

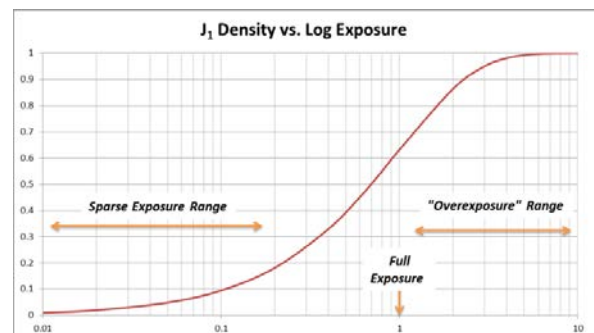


Fig. 2. D-logH exposure characteristic of the Quanta Image Sensor.

¹ Essentially corresponding to a saturation exposure if arrival rate was steady on an instantaneous basis – that is, no shot noise.

As was discussed qualitatively in previous papers [7,8] this D-log H “S-shaped curve” is quite similar to the famous D-log H plot for photographic plate densities following light exposure and development, as reported by Hurter and Driffield in 1890 [9]. In the case of photographic film, it means that film response is non-linear and can tolerate highlights much better than a conventional sensor – a feature desired by classic photographers and cinematographers. For conventional sensors some of this non-linear response can be encoded by using an appropriate value of γ in the postprocessing of the image. Nevertheless, the high end of the non-linear response of film has been hard to match even in HDR solid-state image sensors without introducing artifacts.

II. NOISE

To calculate the noise σ_1 in M_1 , the variance of a binomial distribution needs to be used. In this case we have:

$$\sigma_1^2 = M \cdot P[J_0] \cdot P[J_1] \quad (7)$$

or

$$\sigma_1^2 = \frac{M_0 M_1}{M} \quad (8)$$

or

$$\frac{1}{\sigma_1^2} = \frac{1}{M_0} + \frac{1}{M_1} \quad (9)$$

since $M = M_0 + M_1$. As an example, M_1 and σ_1 were calculated using these formulas and also simulated using Monte Carlo methods for $M = 256$ over 100 trials. The results are shown below in Fig. 3. It can be seen that the noise is suppressed as the illumination approaches full exposure conditions and diminishes quickly for overexposure. In essence, the noise is “squeezed” since the number of empty jots is small, and only these can contribute to a variance in M_1 .

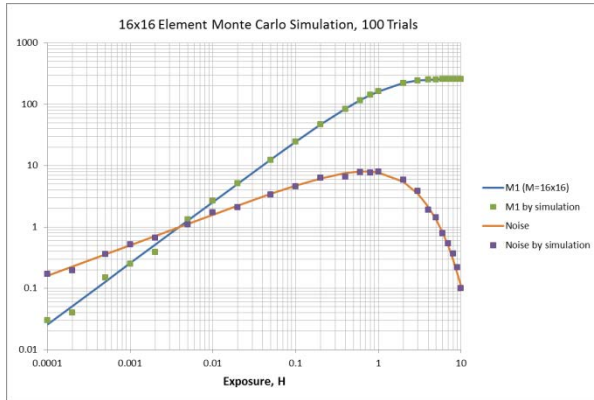


Fig. 3. Signal and noise as a function of exposure

The noise has a maximum value of:

$$\sigma_{1max} = \frac{1}{2} \sqrt{M} \quad (10)$$

which occurs at a total exposure of $H = \ln 2$ or when the jots are half filled ($M_0=M_1=M/2$). It is noted that the noise is already squeezed by a factor of $\sqrt{2}$ from what it would be if one just used the classical shot noise equation, even though the jots are only “half full.”

III. SNR

Signal to noise ratio (SNR) for the QIS can be readily calculated. The signal S is M_1 . The noise is σ_1 and SNR is given by:

$$SNR = \sqrt{\frac{M_1}{1 - M_1/M}} \quad (11)$$

Generally we are more interested in the exposure-referred SNR, SNR_H , since when the noise is squeezed the apparent SNR artificially rises. SNR_H is given by:

$$SNR_H = \frac{H}{\sigma_1 \frac{dH}{dS}} \quad (12)$$

From Eq. 5 we obtain:

$$\frac{dH}{dS} = \frac{1}{M e^{-H}} \quad (13)$$

Thus, SNR_H is given by:

$$SNR_H = \sqrt{M} \frac{H}{\sqrt{e^H - 1}} \quad (14)$$

Fig. 4 shows exposure-referred SNR_H as a function of exposure in the QIS for $M = 4,096$. SNR_H peaks at about 50, or 34 dB at an exposure $1 < H < 2$ and then drops off. For the linear, sparse-exposure range, SNR_H reaches approximately 20 or 26dB. This suggests that operating the QIS in the linear regime may require more than 4,096 jot samples per pixel to achieve good image quality over reasonable exposure latitude.

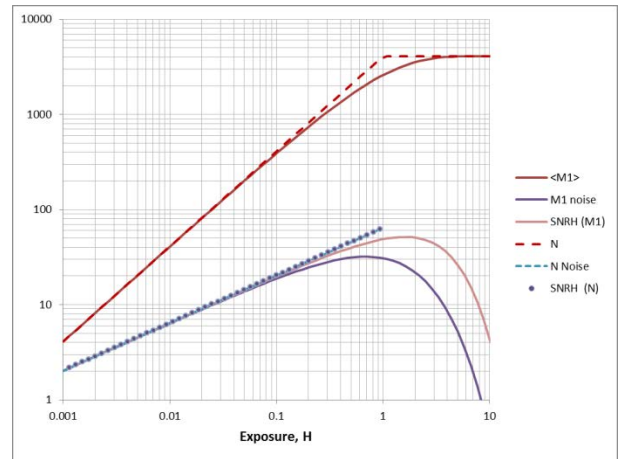


Fig. 4. SNR_H as a function of exposure for $M = 4,096$ jots

The drop off in SNR_H is due to the flattening of the M_1 response so that the same noise would correspond to more uncertainty in exposure, hence lower SNR despite the squeezed noise.

Also shown in Fig. 4 is the hypothetical case where the QIS response is linear ($S=MH$) until it abruptly saturates. The total arrival count N is shown. Its corresponding noise is the classic shot noise \sqrt{N} and since the response is linear, the SNR_H is also equal to \sqrt{N} .

The QIS SNR_H is lower than this hypothetical case by just 2.35 dB at full exposure yet gains over 10 dB in additional exposure latitude (to $\sim 3x$ overexposure).

IV. DYNAMIC RANGE

Dynamic range, DR , is defined as ratio of maximum exposure H_m that just saturates the sensor (or less, depending on linearity requirements) and the exposure-equivalent temporal noise H_n level in the dark.

$$DR = 20 \log \left(\frac{H_m}{H_n} \right) \quad (15)$$

The dark temporal noise consists of read noise entering the column sense amplifier and dark current counts. To calculate its effect we consider the probability distribution of read signals for no photoelectron (state J_0) and with a photoelectron (state J_1) as shown below in Fig. 5.

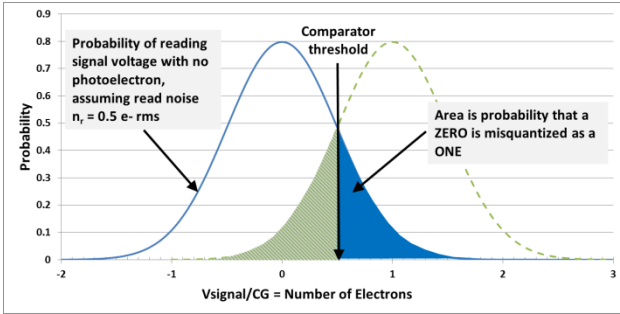


Fig. 5. Read signal probability distributions for states J_0 and J_1 with read noise of 0.5 e- rms.

The comparator threshold is set at 0.5 e- (e.g., 5 mV in the case of 10 mV/e- total gain). The total probability that state J_0 is misread as state J_1 is the solid blue area shown by the arrow. Similarly, the probability that state J_1 is misread as state J_0 is shown by the green hatched area. Integrating the two normal distributions gives us the QIS bit error rate (BER) due to signal chain read noise:

$$BER = \frac{1}{2} \operatorname{erfc} \left(\frac{1}{\sqrt{8} n_r} \right) \quad (16)$$

where n_r is the read noise in e- rms. BER is shown as a function of read noise in Fig. 6.

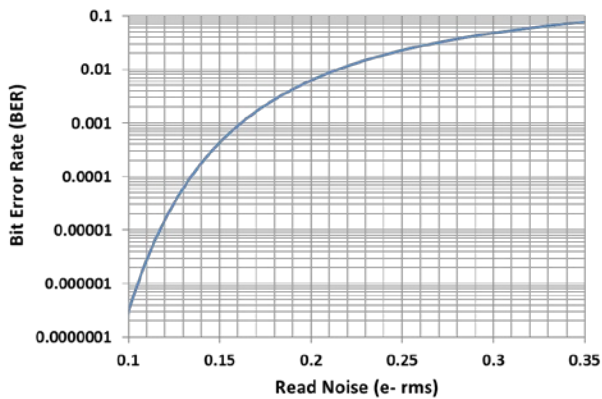


Fig. 6. Bit Error Rate vs. Read Noise for the QIS

From Fig. 6, a reasonable target for read noise n_r is about 0.15 e- rms yielding $BER \approx 1/1000$. For an estimated voltage noise of 150 μ V rms a high conversion gain (e.g., 1 mV/e-) is required. In the dark there is no population of 1's turning to 0's - we are looking for just

0's that turn to 1's, so the same BER value is obtained. For $n_r = 0.15$ e- rms we obtain BER of 0.04% meaning we can expect 1.6 incorrect bits out of every 4096. That corresponds to an exposure-equivalent noise of $H_{nr} = 0.0004$ since $MH_{nr} = 1.6$ for $M=4096$. Generally, then,

$$H_{nr} = \frac{1}{2} \operatorname{erfc} \left(\frac{1}{\sqrt{8} n_r} \right) \quad (17)$$

where n_r is measured in electrons r.m.s. and is less than unity, not including dark current. The noise H_{nr} is a very steep function of n_r . For $n_r = 0.20$ e- rms, H_{nr} grows 14x to 0.006 and for $n_r = 0.10$ e- rms, H_{nr} reduces 1500x to 0.000003. In essence, read noise is likely either to be a large problem, or no problem.

Dark current is difficult to predict, but generally we can expect levels similar to those in present day CMOS image sensors. Jot areas are smaller, voltages lower but field-bunching stronger and doping higher, among other factors. For example, SOA is 15 e-/s at 60C for 1.4 μ m pixel [10]. A jot that is 1/100th the area might reasonably be expected to have a dark current of 0.5 e-/s. For a 16x time-oversampled QIS, the integration interval might be 2 msec so the expected number of dark carriers is 0.001. This is 4 dark bits out of every 4096 and best considered temporal noise. We can assign an exposure-equivalent dark current H_d and set it to 0.001 in this estimate. Note H_d depends on the integration time since exposure is defined as the number of expected arrivals over the integration interval.

The dynamic range of the QIS extends from the greater of H_{nr} and H_d , to about $H_m=5$ according to Fig. 4 yielding a dynamic range of approximately $20 \log (5.0/0.001) = 74$ dB. For good linearity, H_m might need to be as low as 0.1 with a lower dynamic range of 40 dB in this example.

V. HIGH DYNAMIC RANGE

As in conventional CMOS image sensors [11], dynamic range can be improved by combining different integration periods. In Fig. 4 the transfer characteristic and SNR for a QIS device was shown, where nominally the 4096 jot samples could be considered 16x16 in space, and 16 time slices, with all samples having the same integration time. In that case, the SNR peaked at about 51. Instead, just as an example, we could break the 16 time slices up into 4 groups with 4 slices each, each group having a different integration time for the time slices. For example, the first group of slices could have an integration time of 1 (normalized to the readout scan time), the 2nd 0.2, the 3rd 0.04, and the 4th 0.008 - basically 5x difference between each set. The total readout time for the QIS would be the same, but the exposure period would be different for each group.

This example is illustrated in Fig. 7. The signal output from each group (assuming they are summed) is shown as a function of exposure, along with the noise as a function of exposure. The total noise is also shown. Finally, the exposure referred SNR_H is shown.

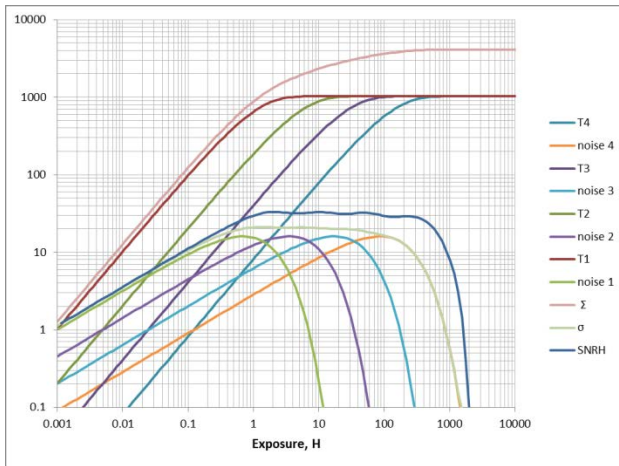


Fig. 7. Illustration of high dynamic range exposure with QIS. Maximum exposure H_m is extended about 125x or 42dB. Exposure-referred SNR_H is relatively flat at about 30 dB.

The maximum exposure has been extended to $H_m \cong 400$, an extension of 80x or 38 dB to a total DR of 112 dB. SNR_H is relatively flat between $H=1$ and $H=400$ with a value of ~ 30 or 30 dB, representing a drop of about -4dB from its single integration interval peak.

Summing the outputs is one way of creating the HDR output signal, but other methods with improved linearity can also be considered, for example by weighting the sum of each group before summing the group signals.

Generally for HDR operation, the shape of the SNR curve and its rough value is traded against DR through the choices of integration intervals and number of slices in each group. Higher DR comes at the expense of lower SNR, at least over some exposure range.

VI. CONCLUSION

The QIS behaves differently than conventional image sensors with respect to exposure linearity and noise characteristics due to the binary nature of jot outputs. Under sparse exposure, the QIS is linear and has SNR characteristics similar to that of conventional image sensors. For higher exposures, the non-linearity can be exploited to yield film-like qualities and high dynamic range with nearly flat exposure-referred SNR.

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VIII. REFERENCES

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